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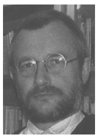
The Exchange Rate and Purchasing Power Parity in Arbitrage-Free Models of Asset Pricing *

by P. SERCU *



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ABSTRACT

Assuming that asset markets are complete and arbitrage-free, the exchange rate can be expressed in terms of observables in a multicountry, multigood general equilibrium economy. In contrast to existing models of the exchange rate, this general model allows for international differences in consumption preferences, time preferences, and the degree of risk aversion, and does not need to specify the imperfections in commodity markets. Changes in the equilibrium exchange rate are given by international differences in: (i) inflation rates computed from marginal spending weights, (ii) growth rates of real spending, weighted by the countries' measures of relative risk-aversion, and (iii) subjective discount rates. The discount rates and risk aversions can vary both over time and across countries. In this general framework, relative Purchasing Power Parity (PPP) holds only if preferences are homothetic and, either (a) investors are risk neutral or (b) commodity markets are perfect and preferences are identical across countries; in all other cases, CPI inflation is only one of the factors determining exchange rate changes. Thus, compared to this general model for exchange rates, standard regression and cointegration tests of PPP suffer from missing-variables biases, errors-in-variables biases, and ignore variations in risk aversions across countries and over time. An attractive feature of this model is that it nests several existing equilibrium models of the exchange rate and also PPP, thus providing a theoretical framework to distinguish empirically between these models. When estimating this equation as a long-run relationship, Sercu and Uppal (2000) and Apte, Sercu and Uppal (2006) find significant evidence against long-run PPP and largely supportive evidence in favor of the more general model.

JEL classification: F31
Keywords: Purchasing power parity, general equilibrium, non-traded goods, regression tests, cointegration

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I. INTRODUCTION

My objective, in this paper, is to review a model of the exchange rate in an arbitrage-free economy characterized by multiple countries and multiple goods, without imposing strong restrictions on output processes, preferences, or the role of money. The model general also allows for various imperfections in commodity markets—non-traded goods, fixed and variable costs for trading goods internationally, and imperfect competition. The only restriction is that asset markets be complete and perfect. The paper builds on Lewis (1995) and Backus et al. (2003). A special case was already published by Sercu, Uppal and Van Hulle (1995). The first-order decomposition of changes in the pricing kernels (and, by extension, in the exchange-rate changes) was first published by Sercu and Uppal (2000), henceforth referred to as SU (2000). Johansen-Juselius cointegration tests using levels of exchange rates, CPIs, and consumption flows appear in the same source. Estimates of the model using a somewhat different technology are forthcoming in Apte, Sercu and Uppal (2006), henceforth referred to as ASU (2006).

One contribution of SU (2000) is to show that in this general economy changes in the exchange rate reflect

- (i) international differences in inflation rates computed from marginal spending weights,
- (ii) differences in growth rates of real spending weighted by the countries' measures of relative risk-aversion,
- (iii) differences in subjective discount rates, and
- (iv) changes in any other state variable that affects the marginal utility of nominal spending. The discount rates, risk aversions, and sensitivities to state variables can vary over time and across countries. This model reduces to Purchasing Power Parity (PPP) under restrictive conditions. Specifically, for PPP to hold, consumption preferences need to be state-independent and homothetic; and, in addition, either investors need to be risk neutral, or consumption preferences must be identical across countries and commodity markets must be perfect.

In order to examine this model empirically, one needs to express the exchange rate in terms of observable variables. Assuming that preferences are state-independent as well as homothetic, and that both relative risk aversion and time preferences are constant (but not necessarily equal across countries), an expression can be derived for the level of the exchange rate that contains PPP as a special case and is easily testable. When estimating this equation as a long-run relation, ASU find significant evidence against long-run PPP and largely supportive evidence in favor of the more general model. The model does not rule out long-run PPP, but improves upon it.

The theoretical findings are useful for two reasons. Deriving PPP as a special case of a more general model allows one to address a richer set of empirical questions than is possible in the conventional PPP tests. For instance, the general model has implications for existing tests of PPP: it implies that standard regression tests of PPP suffer from missing-variables biases, errors-in-variables biases, and ignore variations in risk aversions across countries and over time. In this sense, this theoretical work complements recent work on exchange rates that has mostly concentrated on improving the econometric methodology and extending the data used for testing the PPP hypothesis.[1]

The second contribution of the SU/ASU papers to exchange-rate theory is that its results are quite general and encompass many earlier models of the exchange rate.[2] Existing equilibrium models of the exchange rate often depend on very specific assumptions about the number of goods and countries, utility functions, production technologies, and the type of friction in the international goods markets. The model presented here for exchange-rate changes, in contrast, is one where utility functions are quite general and can differ across countries, and where commodity markets may be imperfect. For example, our model encompasses the standard monetary model of exchange rates and the equilibrium exchange rate models of Stockman (1980), Lucas (1982), Stulz (1987), Dumas (1992), Backus and Smith (1993), Basak and Gallmeyer (1995), Sercu, Uppal and Van Hulle (1995), and Bakshi and Chen (1997). The economy allows for multiple countries and goods, while existing models are typically of a two-country, single-good economy or of an economy with one traded good and one non-traded good per country. Also, risk aversions needs not be constant over time or across countries. The model also permits various imperfections in commodity markets, whereas existing models of the exchange rate typically allow for just proportional transportation costs, or zero costs for one good and infinite costs for the second (non-tradable) good.[3] For example, in this model international shipment of goods may be costly for some or even all of these goods; and the costs, if any, may involve one-time entry costs that limit competition in the goods markets or fixed recurrent outlays (as in Baldwin and Krugman (1989), Dixit (1989) and Krugman (1989)) and/or purely variable expenses (as in Dumas (1992)), possibly with economies of scale.[4]

There are two reasons why the SU/ASU results that are so general. First, if financial markets are complete, one can use the martingale-pricing approach to determine the exchange rate. Thus, there is no need to determine explicitly the equilibrium in the international economy.[5] Second, the model just expresses the exchange rate in terms of observables rather than exogenous factors, which is a much harder task.

The third contribution of SU/ASU papers is the empirical work. Working with the version of the model where the exchange rate can be expressed in terms of observables, and which nests PPP as a special case, we compare the more general model to PPP. SU/ASU find that the additional variables suggested by the model are significant; that is, the general model outperforms PPP. Thus, we reject long-run PPP as the best available model for the real exchange rate in favor of our more general formulation. To take into account small-sample problems in the estimation, the significance tests are based on extensive Monte-Carlo simulations instead of asymptotic distributions.

The rest of the paper is organized as follows. In Section II, I describe the economy, and derive the change in the exchange rate in terms of the arguments of the pricing kernels. In Section III, I express the exchange rate in terms of observable variables under assumptions that are increasingly restrictive. This brings us, in Section IV, to the conditions under which PPP holds, and the implications of our theoretical work for empirical tests of PPP. Empirical results are reviewed in Section V. Section VI concludes. To make it easier to identify the main results, these are presented as propositions with intermediate results in lemmas.

II. THE ECONOMY AND THE EXCHANGE RATE

In this section, I describe a model of a multicountry, multigood economy with imperfect commodity markets, and derive a general expression for the exchange rate change in terms of the arguments of the pricing kernels. I impose only a few (very standard) restrictions on preferences, and none on the production or endowment processes or on the degree or type of commodity market imperfections. Nor do I need to specify explicitly the role money plays in the economy.[6] The main assumption is that asset markets are complete and perfect. I then derive the Lewis (1995) and Backus et al. (2003) general proposition about exchange-rate changes and my decomposition.

A. *The economy*

The economy that I consider consists of $K \geq 2$ countries. The text focuses on two arbitrarily selected countries, called the home country (subscript $k = 1$) and the foreign country ($k = 2$). Each country has a representative consumer, with a standard, strictly quasi-concave utility function defined over $N \geq 1$ goods, and with lifetime expected utility maximization as the objective. Across countries these representative individuals may differ in terms of risk-aversion, consumption preferences, time preferences, and initial wealths.

The outputs of each of the N goods can be stochastic over time. The economies could be exchange economies (with the endowment process given exogenously) or production economies with endogenous investment decisions. The specification of the production technologies or endowment processes is quite general: some goods may be produced everywhere, while other goods may be produced only in some countries. International shipment of these goods may be costly for some or even all of these goods; and the costs, if any, may involve fixed entry costs or fixed recurrent costs and/or purely variable costs, possibly with economies of scale. Given these costs for transferring goods across countries, some goods may be traded all the time, some may be tradable in the strict sense (that is, traded only if the price difference is sufficiently large to justify incurring the shipment costs), and some goods may be *de facto* non-tradable.

As in most general-equilibrium models of the exchange rate, the assumption is that financial markets are frictionless and complete, and admit no arbitrage opportunities. This assumption allows us to derive the exchange rate without having to specify the opportunity set explicitly. Thus, in contrast to models such as Dumas (1992) and Sercu, Uppal and Van Hulle (1995) that solve a planner's problem to characterize the equilibrium, We do not need to assume perfect competition in the commodity and factor markets.

B. *The exchange rate*

Let us first define the marginal indirect utility of nominal spending in country k at date t, Λ_k. Consider the static problem of a consumer who faces prices for the N goods, $\underline{p}_k(t)$, and who wishes to allocate an arbitrary budget of $C_k(t)$ over the consumption of these goods, $\underline{c}_k(t)$, in order to maximize utility, $U_k(\underline{c}_k(t), \underline{X}_k(t), t)$, where $\underline{X}_k(t)$ is a vector of (possibly country-specific) state variables that affect utility.[7] This problem can be written as:

$$V(C_k(t), \underline{p}_k(t), \underline{X}_k(t), t) \equiv$$
$$\text{Max}_{c_{ki}(t)}, \left\{ U_k(\underline{c}_k(t), \underline{X}_k(t), t) - L_k \, [N,\textstyle\sum,j=1 c_{kj}(t)\, p_{kj}(t) - C_k(t)] \right\} ,$$

where

$V(C_k(t), \underline{p}_k(t), \underline{X}(t), t)$ refers to the period-t indirect utility function of total spending, given prices,
$\underline{c}_k(t)$ denotes the vector of consumption quantities $c_{kj}(t)$ of good j (=1, …, N) consumed by the representative individual in country k (=1, …, K) at time t,
$U_k(\underline{c}_k(t), \underline{X}_k(t), t)$ denotes the utility function of the representative investor in country k, and implicitly includes the discounting for time,
$\underline{p}_k(t)$ denotes the vector of local-currency price of good j in country k, $p_{kj}(t)$
$C_k(t)$ denotes the nominal consumption budget, expressed in terms of country k's currency,
$\underline{X}_k(t)$ denotes the vector of M_k state variables that affect the utility of consumption in country k

Thus, the marginal indirect utility of nominal spending in country k is the multiplier in the above optimization problem:

$$\Lambda_k(C_k(t), \underline{p}_k(t), \underline{X}_k(t), t) = \textbf{Fout!}.$$

For notational convenience, $\Lambda_k(C_k(t), \underline{p}_k(t), \underline{X}_k(t), t)$ is often abbreviated into $\Lambda_k(t)$, below.

Let's now assume that financial markets are complete and perfect, and admit no arbitrage opportunities. This allows us to derive the exchange rate without having to specify the opportunity set explicitly. In Lemma 1.1 below, the marginal indirect utility of nominal spending in the two countries is linked to the nominal exchange rate, S(t), defined as units of country-1 currency per unit of currency 2.[8]

Lemma 1.1

In an international economy with complete, arbitrage-free and frictionless capital markets, the change in the nominal exchange rate, S(t+1)/S(t), is given by the ratio of the change in the marginal indirect utility of total nominal spending in the two countries:

$$\textbf{Fout!} = \textbf{Fout!} \qquad (1.1)$$

The level of the nominal exchange rate, S(t), therefore, is proportional to the ratio of the marginal indirect utilities of nominal spending in the two countries:

$$S(t) = \theta_2 \ \textbf{Fout!}, \qquad (1.2)$$

where θ_2 is a constant.

Proof

Let $d_{ki}(t+1)$ be the nominal payoffs from security i in currency k at time t+1, and let the time-t price of this security in terms of currency k be given by $P_{ki}(t)$. In an arbitrage-free and frictionless capital market, the (nominal) price of security i in terms of the currency of country 2 is

$$\begin{aligned} P_{2i}(t) &= E_t \ \textbf{Fout!}, \\ &= E_t \,[\, m_2(t+1)\, d_{2i}(t+1)] \,, \qquad (1.3) \end{aligned}$$

where $m_k(t+1) \equiv \Lambda_k(t+1)/\ \Lambda_k(t)$ denotes the pricing kernel in country 2. In currency 1, a similar equation, $P_{1i}(t) = E_t\ [m_1(t+1)\ d_{1i}(t+1)]$, holds. In addition, in frictionless and arbitrage-free markets the home- and foreign-currency future payoffs are related as $d_{1i}(t+1) = S(t+1)\ d_{2i}(t+1)$, while the home- and foreign-currency current prices are likewise related as $P_{1i}(t) = S(t)\ P_{2i}(t)$. Thus, for any asset i, the home-country pricing equation can be expressed as

$$P_{2i}(t)\, S(t) \ = \ E_t \left[m_1(t+1)\, [S(t+1) \ ¥ \ d_{2i}(t+1)] \right] \ . \qquad (1.4)$$

Equations (1.3) and (1.4) imply the following set of restrictions on the exchange rate:

$$E_t\ \textbf{Fout!} = 0\ , \text{ for all assets i.} \tag{1.5}$$

One solution that satisfies (1.5) is that, in each state at time t+1, the exchange rate change is given by

$$\textbf{Fout!} = \textbf{Fout!}. \tag{1.6}$$

In complete markets there can only be one set of exchange rate changes that satisfy (1.5), so it must be (1.6). Substituting the definition of $m_k(t)$ into (1.6) yields the expression for the change in the exchange rate in (1.1). Lastly, to get (1.2), note that, from (1.1), $S(\tau)$ must be proportional to $\Lambda_2(\tau)/\Lambda_1(\tau)$. The proportionality factor, denoted by θ_2, must be constant since it cancels out in (1.6). *////*

An economic interpretation of θ_2 is offered in Section III.C. To obtain an expression for changes in the exchange rate in terms of inflation and growth in nominal spending, consider a first-order Slutsky decomposition of the changes in the marginal indirect utilities, as in, for instance, Barten (1964) and Breeden (1978). That is, decompose $d\Lambda_k/\Lambda_k$ into the effect of the curvature of the indirect utility (the degree of relative risk aversion) and the effects of changes in each of the arguments of the indirect utility—the nominal spendings, the prices for all goods, and time. To a first-order approximation,[9] the change in the log nominal exchange rate is then given by the sum of at least three terms. The first term captures the international differences in time-preference patterns—the changes in the marginal utility caused by the mere passing of time, holding constant consumption and prices. The second term is the international difference in real consumption growth rates, weighted by each country's relative risk aversion, and the third term is the international difference in marginal inflation rates. If utility is state-dependent, then exchange rate changes reflect, in addition, changes in all relevant state variables. Note that $d\Pi/\Pi$ and $d\pi/\pi$ is just notation for the average and marginal inflation rates; the integrated levels, Π and π, do not generally exist.

Proposition 1.1
The change in the nominal exchange rate, to a first -order approximation, is

$$\textbf{Fout!} = [\delta_2(t) - \delta_1(t)]\ dt\ + \eta_1(t)\ \textbf{Fout!} - \eta_2(t)\ \textbf{Fout!}$$
$$+\ \textbf{Fout!} - \textbf{Fout!} + \textbf{Fout!}\ \zeta_{1s}\textbf{Fout!} - \textbf{Fout!}\ \zeta_{2s}\textbf{Fout!}, \tag{1.7}$$

where

$\delta_k(t) \equiv -\ \textbf{Fout!}$, *the semi-elasticity of marginal utility with respect to time, that is, the measure of instantaneous time preference,*

$\eta_k(t) \equiv -\ \textbf{Fout!}$, *the degree of relative risk aversion,*[10]

$\zeta_{k,s}(t) \equiv \textbf{Fout!}$,*the elasticity of marginal utility with respect to state* *variable* $X_{k,s}$,

Fout! $\equiv$ **Fout!**, *inflation weighted on the basis of total consumption,*[11]

and

Fout! $\equiv$ **Fout!**, *inflation weighted on the basis of marginal consumption.*[12]

Proof

Start from the total differential of $\Lambda_k = \Lambda_k(C_k, \underline{p}_k, \underline{X}_k, t)$ and then substitute the definition $\Lambda_k = \partial V_k/\partial C_k$. In the third line, use the definition of time preference and invoke the property $\partial V_k/\partial p_{kj} = - c_{kj}\ \partial V_k/\partial C_k$ (Roy's Identity). Next, use the rule for differentiating a product and the definition of ζ_{ks}. Finally, bring out the percentage changes in the budget and the prices, rearrange, and use the definitions of relative risk aversion η and of total and marginal inflation:

Fout! = **Fout!**

= **Fout!**dt + **Fout!**

= $-\delta_k(t)$ dt + **Fout!**

= $-\delta_k(t)$ dt + **Fout!**+ **Fout!**□$_{ks}$ **Fout!**

= $-\delta_k(t)$ dt − **Fout!**

− **Fout!**p_{kj} **Fout!**+ **Fout!**□$_{ks}$ **Fout!**

= $-\delta_k(t)$ dt − $\eta_k(t)$ **Fout!**− **Fout!**+ **Fout!**ζ_{ks} **Fout!**. (1.8)

Substitution of (1.8) into (1.6) then immediately produces (1.7). ////

Observe that the change in the exchange rate in our general model, (1.7), contains marginal inflation rates, $d\pi_k/\pi_k$, which are not observable, as well as the changes in unidentified state variables, X_{ks}. In addition, in this very general formulation the time preference parameters, $\delta_k(t)$, the relative risk aversions, $\eta_k(t)$, and the sensitivities of marginal utility to the state variables, $\zeta_{ks}(t)$, are as yet unspecified functions of the variables in the model. In the next section, I specialize the model of the exchange rate and its dynamics into versions that can easily be estimated from the data.

III. THE EXCHANGE RATE EXPRESSED IN TERMS OF OBSERVABLES

In this section, I first impose restrictions on the primitives so as to obtain the exchange rate level and its dynamics in a form that can be estimated empirically. The general exchange rate model in Proposition 1.1 turns out to encompass several recent equilibrium models that assume utility functions of the state-independent/homothetic/constant-relative-risk-aversion/constant time-preference class but restrict the number of goods and countries, and the type of imperfections in the commodity markets. In the next section, we link the general model to classical PPP model and tghe Balassa-Samuelson theories of deviations from PPP.

A. *Homothetic preferences*

To obtain a testable equation, one first needs to eliminate, from (1.7), the marginal inflation rate, for which data are not available. This is achieved by restricting the preferences to homothetic functions: under this assumption, the relative consumption pattern that follows is independent of the level of the total budget. The result is as follows:

Proposition 2.1
With homothetic utility functions, the change in the nominal rates, to a first-order approximation, is given by

Fout!= $[\delta_1(t) - \delta_2(t)]$ dt + $\eta_1(t)$ **Fout!**+ $[1 - \eta_1(t)]$ **Fout!**– $\eta_2(t)$ **Fout!**– $[1 - \eta_2(t)]$ **Fout!**
+ M,Σ,s=1 ζ1s**Fout!**– **Fout!**ζ2s**Fout!**, (2.1)

and the change in the real exchange rate is

Fout!= [δ1(t) – δ2(t)] dt + η1(t) **Fout!**– η2(t) **Fout!**
+ M,∑,s=1 ζ1s**Fout!**– **Fout!**ζ 2s**Fout!**. (2.2)

Proof
With homothetic utility, $p_{kj}(\partial c_{kj}/\partial C_k) = p_{kj}(c_{kj}/C_k)$, implying that $d\pi_k/\pi_k = d\Pi_k/\Pi_k$. Substituting this in (1.7) gives equation (2.1). Equation (2.2) then follows immediately by subtracting the inflation differential from both sides of (2.1) and regrouping, on the right-hand side, the items associated with risk aversion. ////

From (2.2), changes in the real exchange rate reflect differences across countries in real-spending growth corrected for the effect of different impatience rates $\delta_k(t)$, as well as changes in the state variables. It is easily verified that, if utility is state-independent, then $-\delta_k(t) - \eta_k(t)$ $d[C_k/\Pi_k]/[C_k/\Pi_k]$ is just the growth rate of country k's marginal utility of real spending, decomposed into a part explained by time preferences and a part that reflects changing real consumption. However, it is important to realize that real spending is endogenous, and therefore, depends on, among other things, risk aversion and time preference.

B. *The exchange rate change under state-independent, homothetic, and constant-coefficient utility*

The problems of unspecified state variables and time-varying time preferences are eliminated under the assumption of state-independent (SI) and time-additive lifetime utility with constant time preference (CTP): that is, lifetime utility $\sum^{T,s=t} U(\underline{c}_k(s), \underline{X}_k(s), s)$ is of the form $\sum^{T,s=t} \exp(\delta_k s)\ u(\underline{c}_k(s))$. Under the additional assumption of constant relative risk aversion (CRRA) utility functions, time variation in the ηs is also eliminated. This leads to the following result.

Proposition 2.2
With homothetic constant-coefficient utility functions, the change in the nominal exchange rate, to a first -order approximation, is

Fout!= $[\delta 1–\delta 2]$ dt + $\eta 1$ **Fout!**+ $[1–\eta 1]$ **Fout!**– $\eta 2$ **Fout!**– $[1–\eta 2]$ **Fout!**. (2.3)

Proof
Time-additivity and state-independence imply that $\Lambda_k(C_k, \underline{p}_k, \underline{X}_k, t) = \exp(-\delta_k t)\ \Lambda^*{,}_k(C_k, \underline{p}_k)$. where $\Lambda^*{,}_k(C_k, \underline{p}_k)$. is the undiscounted marginal utility. Therefore, our earlier measure of time preference simplifies to the constant δ_k:

$\delta_k(t) \equiv$ **Fout!**= δ_k.

Making this substitution in (2.1) and assuming that $\eta_k(t)$ is a constant, we obtain equation (2.3). ////

Thus, with state-independence, constant time preference (CTP) and CRRA the intercept and slope coefficients in the real exchange rate model (2.2) are constants. This has obvious advantages for empirical tests on first-differenced logarithmic data. However, the empirical literature on PPP increasingly relies on cointegration analysis using data in levels rather than in first-differenced form. To relate our model to tests of PPP based on the levels of exchange rates and of price indices, we need an integrated version of (2.3). This is provided in the next section.

C. *The level of the exchange rate in the homothetic, state-independent, constant-coefficient model*

To obtain the level of the exchange rate it is convenient to start directly from Lemma 1.1 rather than from (2.3).[13] I first derive the general implications of constant time preference and homotheticity for the level of the exchange rate, and I then impose constant relative risk aversion.

As mentioned before, with time-additive and state-independent utility functions and a constant discount rate, lifetime utility $\sum^{T,\ t} U(c_k(s), s, t)$ is of the form $\sum^{T,s=t} \exp(-\delta_k\ s)\ u(c_k(s))$. When, in addition, utility is homothetic, the period-by-period utility function $u_k(c_k(t))$ can be written as $\Phi[\upsilon_k(\underline{c}_k(t))]$, where $\upsilon_k(\underline{c}_k(t))$ is linear homogenous in the consumption quantities and Φ_k is a positive, monotone (and, usually, concave) transformation. The function $\upsilon_k(\underline{c}_k(t))$ can be thought of as summarizing the consumption preferences (which, for homothetic functions, are independent of wealth or total spending), while the curvature of the transformation, $\Phi(.)$, reflects the degree of risk aversion. This separation of consumption preferences from risk aversion makes it possible to characterize the level of the exchange rate in terms of the level of nominal spending, the price level and relative risk aversion, and possibly a time trend.

If the function $\Phi[\upsilon_k(\underline{c}_k(t))]$ is at its maximum value given a consumption budget constraint, then $\upsilon_k(\underline{c}_k(t))$ must also be at its maximum value subject to the same constraint. It is well known (see, for instance, Samuelson and Swamy (1974)) that the solution of the linear-homogenous problem,

$$v_k(C_k(t),\ \underline{p}_k(t)) \equiv \operatorname*{Max,}_{c_{kj}(t)} \quad \{\upsilon_k(\underline{c}_k(t)) - \lambda_k(t)\ [\textstyle\sum c_{kj}(t)\ p_{kj}(t) - C_k(t)]\}\ , \tag{2.4}$$

is of the form $v_k(t) = C_k(t)/\Pi_k(\underline{p}_k(t))$, where $\Pi_k(\underline{p}_k(t))$ is independent of nominal spending, $C_k(t)$, and is linear homogenous in the prices.[14] Accordingly, $\Pi_k(\underline{p}_k(t))$ is interpreted as the price level in country k, and $v_k(t) = C_k(t)/\Pi_k(\underline{p}_k(t))$ as total real spending. These properties of homothetic functions lead to the following result:

Lemma 2.1
With homothetic utility functions and constant subjective discount rates, the level of the nominal exchange rate, S(t), is given by

$$S(t) = \theta_2 \exp[(\delta_1 - \delta_2)t]\ \textbf{Fout!}. \tag{2.5}$$

Proof
Using the relations $V_k[C_k(t), \underline{p}_k(t),\ t] = \exp(-\delta_k\ t)\ \Phi_k(v_k(t))$ and $v_k(t) = C_k(t)/\Pi_k(t)$, we can specify the marginal indirect utility of nominal spending

as follows:

$$\textbf{Fout!} \;=\; \exp(-\delta_k\, t)\; \textbf{Fout!} \qquad (2.6)$$

$$=\; \exp(-\delta_k\, t)\; \textbf{Fout!}. \qquad (2.7)$$

Substituting (2.7) into (1.2), we obtain (2.5). ////

Let us now interpret the constant θ_2 and the trend $\exp[(\delta_1–\delta_2)t]$ in equations (1.2) and (2.5). Observe that equation (2.5) implies that the real exchange rate is the marginal rate of substitution for an international preference ordering of the form $\exp(-\delta_1 t)\Phi_1(v_1(t)) + \sum^{K,k=2} \theta_k \exp(-\delta_k t)\Phi_k(v_k(t))$, which would be the objective function if one were using the central-planner’s approach to determine exchange rates (see, for instance, Dumas (1992) and Sercu et al. (1995)). Thus, θ_k corresponds to the weight assigned by the central planner to the utility of country k relative to that of country 1 and determines how world output will be shared internationally, given the opportunity set. It follows that in a decentralized economy with complete markets, θ_k will depend on the factors that determine the international allocation of consumption—the initial wealths of the two countries, which depend, in turn, on the initial endowments and the utility functions. The role of $\exp[(\delta_1–\delta_2)t]$, then, follows immediately. Different impatience factors mean that the two countries are depleting their wealths at different rates. Thus, the proper interpretation of $\exp[(\delta_1–\delta_2)t]$ is that it adjusts the initial θ_2 so as to capture this divergence of the two countries' wealths. Stated differently, $\exp[(\delta_1–\delta_2)t]$ reflects one of the causes of divergence between *undiscounted* marginal utilities, $d\Phi_k/dv_k$, and the model says that differences between undiscounted marginal utilities affect the real rate only if they are not the reflection of heterogeneous time preferences. Thus, the model does not predict a time trend in the exchange rate (as (2.5) may seem to suggest). Rather, the role of the time trend is to correct the ratio of undiscounted marginal utilities for divergences that merely reflect differences in time preferences.

Given that the marginal utilities of aggregate real spending are not observable, equation (2.5) is still not in a form where it can be used to study the empirical behavior of the *level* of the nominal (and real) exchange rates; one needs to make the additional assumptions that investors have power utility functions, with constant relative risk aversion given by $\Box_k$. This allows one to link, in a tractable way, the marginal utilities of real consumption, to real consumption quantities for which data is available. Specifically, with power utility, the exchange rate becomes a loglinear function of both the price level and the level of nominal spending in the two countries, with the constraint that the elasticities of each country's price level and nominal spending sum to unity.

Proposition 2.3
With homothetic preferences, constant subjective discount rates, and constant relative risk aversion, the nominal exchange rate is

$$S(t) = \theta_2 \exp[(\delta_1 - \delta_2)\, t]\textbf{Fout!}, \quad (2.8)$$

where κ_k *equals* $1-\eta_k$ *when* $\eta_k \neq 1$, *and unity otherwise. The corresponding real exchange rate is*

$$S(t)\ \textbf{Fout!} = \theta_2 \exp[(\delta_1 - \delta_2)\, t]\textbf{Fout!}. \quad (2.9)$$

Proof
The power or constant relative risk aversion (CRRA) utility functions have the form

$$\Phi_k(v_k(t)) = \textbf{Fout!}.$$

For $\eta_k \neq 1$, substituting the power utility in (2.5) gives:

$$S(t) = \theta_2 \exp[(\delta_1 - \delta_2)\, t]\ \textbf{Fout!}.$$

Equation (2.8) then follows upon substituting $v_k(t) = C_k(t)/\Pi_k(t)$, and simplifying the resulting expression. The proof for $\eta_k = 1$ is analogous. ////

In the remainder of this paper, we refer to the model with state-independent and homothetic utility with constant time preferences and constant relative risk aversions as the standard power-utility (SPU) model.

D. *Relation to existing models of the exchange rate*

Proposition 2.3 encompasses many existing models of the exchange rate, that have typically been derived in settings with one or two goods (whereof at least one good is tradable only at a cost), equal impatience, and constant relative risk aversion. For example, the special version of equation (2.8) with $\eta_1 = \eta_2$ and $\delta_1 = \delta_2$ has been obtained by Sercu, Uppal, and Van Hulle (1995) assuming two countries, and one (imperfectly tradable) good, while Backus and Smith (1993) derive a similar model for the case of CES consumption preferences defined over one perfectly tradable good and one non-tradable good. Stulz (1987) likewise derives (2.9) from a two-country production economy with log investors ($\eta_1 = \eta_2 = 1$) that have identical δ_k's and identical Cobb-Douglas preferences defined over a perfectly tradable good and a non-traded good.[15] Thus, all these special versions generalize to cases where there are N goods (regardless of their degree of tradability) and K countries, and where the degree of relative risk aversion and time preference, as well as the commodity preferences, can differ across countries.

The model also nests the monetary economies considered in Bakshi and Chen (1997) and Basak and Gallmeyer (1996), where the focus is on determining the prices of financial securities rather than expressing the exchange rate in terms of observable variables and relating it to PPP. Bakshi and Chen characterize the exchange rate and prices of financial assets in terms of exogenous variables in an exchange economy where money is neutral. However, they restrict utility to log functions and the endowment processes to lognormal distributions; also, they consider only an equilibrium with perfect pooling, that is, without deviations from PPP. Basak and Gallmeyer (1996) characterize the exchange rate in an exchange economy with money in the utility function. In contrast to these models, there is no need to specify the role of money in the above model.

To conclude this section, note that the equilibrium approach is also compatible with the Balassa (1964) and Samuelson (1964) exchange rate models, that link deviations from PPP to differences in productivity. These models assume that there is a non-tradable good (denoted, below, as good 0) in each country, and one perfectly tradable good (good 1). A price index—whether a real-world pragmatic measure such as a CPI or WPI, or a theoretical index induced by a particular linear-homogenous commodity preference function $\upsilon_k(c_{k0}, c_{k1})$—is linear homogenous in the individual goods prices. Using this homogeneity property and commodity price parity for the traded good, the real exchange rate can then be written as a function of the relative prices of the non-traded good abroad and at home:

$$\begin{aligned} S(t)\ \textbf{Fout!} &= S(t)\ \textbf{Fout!} \\ &= \textbf{Fout!}. \end{aligned} \qquad (2.10)$$

Combining (2.10) with propositions about wealth-related differences in spending patterns and technology-related differences in productivities over time and across countries and sectors, Balassa and Samuelson provide an explanation of the empirical regularity that currencies of more developed economies tend to be overvalued by PPP standards.[16] But (2.10), and therefore also the Balassa-Samuelson propositions based on (2.10), is perfectly compatible with the equilibrium approach. The intuition is that the general results rely on asset-pricing, without making any assumptions about the production side of the economy. For this reason, these results must be compatible with any specification of the economy's production side provided the latter is compatible with complete and perfect asset markets.

Corollary to Lemma 2.1
The real exchange rate given in (2.10) is the same as the exchange rate in a complete-markets equilibrium model with a non-traded and a perfectly traded good. Thus, the Balassa-Samuelson explanations for deviations from PPP are compatible with the equilibrium approach to exchange rates.

Proof

For the perfectly traded good (good 1), by assumption the market is frictionless and perfectly competitive, so the allocation of world output of that good must be Pareto optimal. Thus, this allocation must be the solution to a problem of the form

$$\textbf{Fout!}\exp[-\delta_1 t]\ \Phi_1[\upsilon(c_{10}(t)\ c_{11}(t))] + \gamma_2(t)\ \exp[-\delta_2 t]\ \Phi_2[\upsilon(c_{20}(t)\ c_{21}(t))]$$
$$\text{s.t. } \textbf{Fout!} = -1, \qquad (2.11)$$

where $\gamma_2(t)$ is a relative weight that reflects the international distribution of "income" or, more precisely, consumption spending. In a complete capital market, the allocation of consumption of good 1 in each time-state, given the output vector and the shipping costs, must be entirely determined by the agents' *initial* wealths; that is, the weight $\gamma_2(t)$ in (2.11) must be a constant that reflects the initial endowments. Setting $\gamma_2(t) = \theta_2$, the first order conditions of (2.11) are

$$\exp[-\delta_1 t]\ \textbf{Fout!} = \theta_2\ \exp[-\delta_2 t]\ \textbf{Fout!}.$$

This can be re-arranged so as to isolate, on the right hand side, the real exchange rate as identified in Lemma 2.1:

$$\theta_2 \exp[(\delta_1 - \delta_2)\ t]\ \textbf{Fout!} = \textbf{Fout!}\cdot$$

From the individual budget allocation problem (2.4) we know that $\partial\upsilon_k(.)/\partial c_{k1} = \lambda_k(t)\ p_{k1}(t)$; in addition, as already shown in footnote 14, a homogenous function $\lambda_k(t)$ can be identified as $v_k(t)/C_k(t)$, which, in turn equals the purchasing power, $\Pi_k(t)^{-1}$. Thus, the equilibrium condition is

$$\theta_2 \exp[(\delta_1 - \delta_2)\ t]\ \textbf{Fout!} = \textbf{Fout!}, \qquad (2.12)$$

which is identical to the right hand side of (2.10). ////

IV. PURCHASING POWER PARITY

In this section, we first relate the general model of Proposition 1.1 to the PPP view of exchange rates and show that PPP holds only under very restrictive conditions. Relative PPP holds when

$$S(t) = \Psi\Pi_1(t)/\Pi_2(t)\ \forall\ t \iff \textbf{Fout!} = \textbf{Fout!}, \qquad (3.1)$$

while Absolute PPP holds when, in addition, the constant Ψ equals unity. Proposition 3.1, below, identifies the alternative sets of assumptions under which equation (3.1) is true:

Proposition 3.1
Two alternative sufficient sets of assumptions for PPP are:

(i) *commodity markets are frictionless* and *agents have identical, homothetic consumption preferences—irrespective of their time and risk preferences. Then, also* $\Psi = 1$ *(Absolute PPP holds).*

(ii) *agents have linear homogenous utility functions* ($\eta_k = 0$) *and equal impatience across countries* ($\delta_1 = \delta_2$)*—irrespective of market imperfections and international differences in consumption preferences.*

Proof
To prove part (i) of the proposition, note that under the assumptions of frictionless markets, relative prices are equal all over the world. Given identical and homothetic utility functions, it follows that the consumption bundles have the same (relative) composition across countries: at any time t there is but one composite good in the world, with time-varying composition proportional to the aggregate consumption amounts of the individual goods. The quantities of this aggregate good consumed per country are just the $v_k(t)$'s. Equation (2.5) implies that the real exchange rate is the marginal rate of substitution along an indifference curve $\Phi_1(v_1(t)) + \theta_2\ \Phi_2(v_2(t))$. As the composite good can be transferred internationally costlessly, this marginal rate of substitution of $v_1(t)$ for $v_2(t)$ always equals unity. This finishes the proof of Part (i). Part (ii) follows immediately from (2.5) by setting $\Phi(v_k(t)) = v_k(t)$ and $\delta_2 = \delta_1$. ////

From the above proposition, we see that PPP is a special case of the SPU model: PPP holds if the time-preference and η-related terms on the right hand side of (2.1) vanish. In case (ii) of Proposition 3.1, the terms η_k and $(\delta_2 - \delta_1)$ are zero by direct assumption about the utility functions: investors are postulated to be risk neutral and have identical time preferences. Interestingly, this result for risk-neutral economies, already noted by Dumas (1992) in a single-good model with CRRA utility functions, does not in any way depend on arbitrage in the goods markets. For the more familiar commodity-arbitrage-based case (i), in contrast, the terms in η and δ cancel out across countries: under the assumptions of perfect markets and identical homothetic consumption preferences, there exists one common and perfectly tradable composite good; so the marginal utilities of real spending are equalized across countries. Specifically, in (2.9), $\delta_2^t\ d\Phi_2(t)/dv_2(t)$ always equals $\delta_1^t\ d\Phi_1(t)/dv_1(t)$, so that PPP obtains.

VI. EMPIRICAL TESTS OF PPP AND THE STANDARD POWER UTILITY MODEL

In this section, I briefly review the SU/ASU tests of the SPU (standard power utility) model in (2.8)-(2.9). The SPU model of the exchange rate, including PPP as a special case, specifies relations between levels of endogenous variables. As such, the model should be tested using techniques like GMM, instrumental variables, or cointegration analysis.[17] the SU/ASU approach is similar to what has become standard in the empirical literature on PPP: rather than requiring that equation (2.8) hold exactly at any given date, they verify whether the variables identified in the model have an influence on the exchange rate in the long run. As they succeed in doing so, they also reject PPP—not necessarily in the sense that real exchange rates are non-stationary, but in the sense that PPP does not provide the best possible explanation of long-run exchange-rate behavior.

Thus, in the tests the PPP error-correction-model is extended by introducing the additional variables suggested by the SPU model: real spendings, and a time trend to allow for divergence between the real consumptions across countries caused by differences in international time preferences. I first summarize SU's standard tests of the cointegration type. I then turn to ASU, who use regression analysis to compare the performance of the SPU model relative to PPP (which is nested in the general model) using the regression specification based on results in Phillips and Lorethan (1991). ASU do extensive Monte-Carlo simulations on the distribution of the t-statistics under three alternative data-generating models that all exclude any role for real consumption but differ in their assumptions about the relation between exchange rates and prices. They find that the SPU model outperforms PPP.

The data used in the above analyses are quarterly consumption spending series, CPI data in the last month of the quarter, and end-of-quarter exchange rate data from IFS for the United States (US), Japan (JP), Germany (DE), the United Kingdom (UK), and Switzerland (CH), over the period 1974:I to 1994:IV.[18] We take the USD as the reference currency (currency "1", in the theoretical part) and convert all exchange rates into USD per unit of foreign currency. In what follows, the other country is generally referred to as country k = {DE, JP, US, CH}.

A. *ADF and Johansen-Juselius Trace Tests of Sercu & Uppal (2000))*

ADF tests for unit roots and Johansen-Juselius tests for the number and properties of cointegration relations are natural preliminaries for the regression analysis, for a number of reasons. Unit roots are the motivation for ASU's adoption, in Section VI.B, of the Phillips-Lorethan (1991) specification of the regression and the Monte-Carlo based confidence intervals. The Johansen-Juselius tests for the presence of cointegration confirm that the long-run relationship we are estimating in Section VI.B is really there. Lastly, the Johansen-Juselius tests of the hypothesis that the long-run coefficients conform to PPP provide additional support for our regression-based evidence against the PPP hypothesis.

First, the Augmented Dickey-Fuller (ADF) statistics never reject the hypothesis that the data have unit roots cannot be rejected for any of the price, consumption, and nominal-rate series.[19] The next step is to estimate the number of cointegration relations in the data, using the Johansen-Juselius maximum-eigenvalue (λ_{max}) and Trace statistics. These tests are carried out on both nominal and real data sets. The tests on nominal data in each of the bilateral data sets, the US versus country k, provide convincing evidence of at least one cointegration relation for all countries except Germany; for Japan, there even seem to be at least two such relations. Proceeding under the assumption that there is at least one long-run relation per country pair, and two for the JP-US data, SU next test whether it is possible that four of these relations are the PPP ones, with unit parameters for lnS, $\ln P_{us}$, and $-\ln P_k$. All tests, corrected for small-sample effects following Richards (1995), reject PPP.[20] This failure of PPP confirms the findings of others (see, for instance, Nessén (1994),[21] and Froot and Rogoff (1994)). The finding that the βs differ from unity already suggests that prices may have been proxying, to some extent, for a missing variable.

The tests on real data (that is, real exchange rates and real consumptions) provide additional evidence against PPP, as follows. First, tests on the five real-consumption data series, excluding exchange rates, do not reveal any long-run relation between these data. This implies that any cointegrating relation found in a set that includes real exchange rates must link the real rate to at least one of the real-consumption series, which then tells us that PPP is not the best possible long-run model for the real exchange rate. The findings from bilateral tests (data for each country k and the US) are as follows. For Germany, the Trace and λ_{max} statistics for the hypothesis of no long-run relations are very close to the 10% critical value (the norm suggested by Johansen and Juselius). For Switzerland and the UK the bilateral tests clearly reject the absence of any cointegration vector in the data. Only for Japan do the bilateral tests provide no strong evidence of a long-run relationship among the real data. Thus, for two or three of the four country pairs there is good evidence that real exchange rates are linked to at least one of the real-consumption variables. In the next section, we verify whether the parameters of the long-term relationship are consistent, in sign and magnitude, with what one would expect under the standard power-utility model.

B. *Direct Estimation of the SPU Model (Apte, Sercu and Uppa (2006))*

To test the SPU version of the equilibrium approach, one cannot use directly the Johansen and Juselius (1992) and Horvath and Watson (1993) tests because, unlike PPP, the SPU model does not specify exactly the long-run cointegration vector. That is, while PPP implies that, in a nominal model, all βs should equal unity, the SPU model says only that the coefficients in (3.4) are time-preference and risk-aversion parameters, and because our knowledge about these parameters is, at best, sketchy, there no longer are any specific hypotheses about the long-run coefficients. Thus, ASU follow a more conventional regression approach: estimating the coefficients, checking for the right sign, and testing whether they differ significantly from zero. For the estimation, they use a regression specification based on the work by Phillips and Lorethan (1991),

$$\ln \textbf{Fout!} = \alpha + \tilde{\beta}_0\, t + \beta_k \ln \textbf{Fout!} - \beta_{US} \ln \textbf{Fout!} \qquad (4.1)$$
$$+ \rho\, \textbf{Fout!}$$
$$+ \sum_{L=us,k} \sum_{l=1,2} \Delta_{t-l,t-l-1}\, \textbf{Fout!} + \sum_{l=1,2} \Delta_{t-l,t-l-1}\, \textbf{Fout!} + \varepsilon$$

where $\Delta_{t,t-l}X(t) = X(t) - X(t-l)$. The first line in (4.1) is the model, stated in levels. The second line captures the first-order autocorrelation, ρ, in the deviations from the long-run equilibrium. The third line adds lagged changes in the relevant variables to pick up any remaining traces of predictability in the error. To estimate (4.1) ASU use Seemingly Unrelated Nonlinear Least Squares, first in bilateral estimations (reported in Table 1) where no restrictions are imposed, and then in joint estimations where they restrict the value of β_{US} (the estimator of η_{us}) to be the same across all the countries.

Rather than relying on asymptotic normality, they evaluate the significance of the t-statistics by means of Monte-Carlo experiments. In all of the experiments, price and consumption variables are generated on the basis of estimated VARs, because Johansen-Juselius tests do not reveal any relations among the five price series nor among the five real-consumption series separately. Specifically, the five inflation rates and real-consumption growth rates are first estimated, and then simulated, as mutually correlated ARIMA processes. The simulated exchange-rate data, in contrast, are produced by three alternative "null" data-generating processes, none of allows a role for real consumption or a time trend. These three processes are:

(i) ARIMA: the real exchange rates are assumed to be non-stationary, and follow mutually correlated ARIMA (2,1,0) processes (Roll (1977)).

(ii) PPP: the real exchange-rate equation has, in addition to the VAR part, an error-correction term that links the exchange rate to its PPP value. Again, the innovations are correlated across exchange rates.

(iii) Generalized PPP: the procedure is the same as in the previous model, except that now the coefficients δ_{US} and δ_k in the error-correction term for the nominal exchange-rate equation, $\zeta\ [S(t-1) + \delta_{US}\, \Pi_{US}(t-1) - \delta_k\, \Pi_k(t-1)]$, are estimated rather than pre-set at unity. The motivation for this is that there may be omitted variables in the PPP model, and these variables may be correlated with the CPIs so that $\delta_k \neq 1 \neq \delta_{US}$. However, if the omitted variables are the real consumptions, then the estimates that do include real consumption would still reject the generalized-PPP model.

For each data-generating process ASU simulate 3000 complete 90-quarter, five-country samples (prices, consumptions, and exchange rates), and on each of these samples ASU run (4.1), either bilaterally or with the constraint that β_{us} (the estimator of η_{us}) be identical across equations. They retrieve the t-statistics for each coefficient, rank them, and extract simulated percentile values. As the test on the coefficient estimating the time preference parameter is two-sided and as the estimates of η have occasionally the wrong sign, ASU provide values for the 1st, 5th, 10th, 90th, 95th, and 99th percentiles.

Relative to the standard critical t-values, ASU notice systematically thicker tails in the Monte-Carlo output. Across models, the "Generalized PPP" model tends to generate somewhat wider distributions, while across coefficients the thick tails are especially pronounced for the time-trend coefficient. For all countries the deviations from the estimated long-term relation are quite persistent, with ρ being around 0.8. This implies a half-life of four to five quarters, which is, encouragingly, much lower than the half-life of PPP-deviations estimated in Abuaf and Jorion (1991), three to four years. In the bilateral estimations, six out of eight η-estimates are positive; of the two negative estimates, only one (for German real consumption in the DEM equation) is significant. The average risk-aversion coefficient, 1.66. The time trend is clearly significant for the DEM and GBP, and weakly so for the CHF.

The above estimates in Table 1 ignore our prior that the US relative risk-aversion coefficient should be the same across equations. Monte-carlo simulations show that the data is easily compatible with a common coefficient for the US. When this common β_{US} is imposed across equations, the average estimate of relative risk aversion across the five estimates is virtually unaffected (at 1.67). While the estimated coefficient for German real consumption remains significantly negative, the UK estimate changes sign and becomes positive, and three out of the four time-trend coefficients are now significant. In short, in line with the conclusions from the previous section, we do find evidence that long-run PPP does not provide the best explanation for the equilibrium value of the exchange rate; both the time-trend and risk-aversion have noticeable effects, and in all cases but one the estimated risk-aversion parameter has the correct sign.

Our conclusions differ from the ones obtained by Nissen (1997) and Koedijk et al. (1996): in all our tests, we consistently reject PPP, but find evidence that a time trend and spending data matter—while Nissen (1997) and Koedijk et al. (1996) do not PPP and find no evidence that spending has an effect. Many differences in the test designs may have contributed to these conflicting conclusions. First, the ASU regression specification differs from the regression tests in Nissen and Koedijk *et al.*, but this turns out to be unimportant. A second, and more crucial, difference is that ASU do allow for different impatience factors and risk aversions. Third, the ASU sample is restricted to countries that (with the exception of Japan, prior to 1982) did not have pervasive exchange controls for most of the period. In contrast, the sample in Nissen and Koedijk *et al.* contains many countries that had substantial exchange controls for most of the post-1973 period (Italy, France, and especially South Africa, Spain, and the Scandinavian countries). As the equilibrium model, including the special case obtained under SPU, assumes perfectly integrated capital markets, the presence of restrictions on capital movements is likely to load the dice against the model. Interestingly, in Koedijk *et al.* (1996), PPP does seem to do less well for the countries studied in our paper.

VI. CONCLUSION

Much of the literature on exchange rate determination is based on PPP, with PPP being justified on the basis of the consumption opportunity set (frictionless commodity arbitrage). In contrast, the standard micro-economic equilibrium paradigm views relative prices—and, hence, also exchange rates—as determined not just by consumption opportunity sets, but also by marginal utilities. Focusing on the marginal-utility aspect, the exchange rate can be characterized in a general-equilibrium economy with imperfect commodity markets but complete and frictionless capital markets.

On the theoretical front, in general, real exchange rates are related to differences in initial wealths and time preferences, and also to differences in marginal utilities of total nominal spending. The changes in the exchange rate can, therefore, be related to

(i) differences in growth rates in real consumption weighted by relative risk aversion,

(ii) differences in inflation computed on the basis of marginal spending weights,

(iii) differences in time preferences, and

(iv) other state variables that affect marginal utility. In the special case of homothetic and state-independent utility functions with constant relative risk aversion and time preference, one obtains a closed-form expression for the level of the exchange rate. This model implies that there are missing variables in the PPP equation (the nominal spendings in the two countries and possibly also a time trend), and that the *ceteris paribus* effect of higher domestic prices is a drop in the value of foreign currencies rather than a rise (as predicted by PPP). Unlike related general equilibrium models of the exchange rate, these results are derived without assuming that the degree of risk aversion, the rate of time preference, or preferences over consumption goods are identical across countries.

(v)

Sercu-Uppal (2000) and Apte, Sercu and Uppal (2005) use cointegration techniques to test the equilibrium exchange rate model assuming homothetic, state-independent power utility with constant time-preference parameters. When spending data are excluded from the model, as in standard cointegration tests of PPP, they reject PPP. Direct estimation of the model further reveals that real spending and international differences in time preferences do influence the equilibrium exchange rate.

NOTES

1. For example, Abuaf and Jorion (1990) use Dickey-Fuller tests to establish the presence of mean-reversion in real exchange rates, a phenomenon not evident from autocorrelations tests; and Johansen and Juselius (1992) refine the cointegration techniques in an attempt to find long-run relations between exchange rates and relative price levels that may be hard to detect in first-differenced data. Edison, Gagnon, and Melick (1997) show how the power of these tests can be improved using the Horvath and Watson (1995) procedure. Expanded data sets have been considered by, for instance, Frankel and Rose (1996), Froot, Kim and Rogoff (1995), Lothian and Taylor (1995), Wei and Parsley (1995), and Taylor (1996). Related work includes Engel, Hendrickson and Rogers (1996) and O'Connell (1996). A review of the empirical literature on PPP is provided by Froot and Rogoff (1995), Nessén (1994) and Rose (1996/7).
2. For a recent review of macroeconomic models of the real exchange rate, see Devereux (1997).
3. Empirical support for the effects of shipment costs has been documented in Engel (1993), Rogers and Jenkins (1995), and Wei and Parsley (1995), who find that a significant proportion of the total variation in the real exchange rate arises from deviations from the Law of One Price (LOP). Also, Engel and Rogers (1995) show that within-country deviations from LOP are much smaller than cross-country deviations.
4. Other frictions could be introduced, like shipment lags (goods sent from one country at time t arrive only at time t+1) and transaction lags (a trade arranged at time t is implemented at time t+1 only). It can be shown that neither transaction lags nor shipment lags affect any of our conclusions.
5. The martingale pricing approach has been used to study the relation between exchange rates and international interest rates by Nielsen and Saá-Requejo (1993), Backus, Foresi and Telmer (1996), and Hollifield and Uppal (1997). See Duffie (1992) for details on this approach to asset pricing.
6. With money in the utility function, money balances should be interpreted as just an extra good, with price given by the interest rate, and total spending then defined as spending on goods plus the cost of holding money balances. On the other hand, in the presence of a transaction technology, the goods prices should include the cost of holding money, and so should the measure of total spending.
7. The optimal level of $C_k(t)$, itself, would be obtained by solving the intertemporal problem of the consumer.
8. Lemma 1.1 is a familiar result in the forward-bias literature—see, for instance, Lewis (1995). See Serrat (1996) for a similar application in a continuous-time framework.
9. A (second-order) Ito expansion shows that, in a model with continuous time and stochastic output processes, there will be a drift added to the right hand side of (7) that depends on the risk aversions and the (co)variances of the nominal spendings, the marginal inflation rates, and the total inflation rates.
10. This definition of relative risk aversion, also adopted by Breeden (1978), is a 'real' measure of relative risk aversion because, when taking partial derivatives with respect to C_k, we hold constant the prices. In the one-good case, this definition is identical to the standard definition, $-c_k\,[\partial^2 U_k/dc_k^2]/[\partial U_k/dc_k]$.

11. When money is in the utility function, the interest rate will be part of the price index.
12. The marginal weights, $[\partial c_{kj}/\partial C_k]\, p_{kj}$, sum to unity by virtue of the budget constraint. For notational convenience, we denote the two inflation rates by $d\pi_k/\pi_k$ and $d\Pi_k/\Pi_k$, but we do not wish to imply that the integrated counterparts π_k and Π_k always have known closed-form solutions.
13. Under certainty, the standard calculus adopted thus far would be adequate; hence, equation (2.3) would immediately induce an expression for S(t) that is loglinear in the levels of the price indices and the consumption spendings. However, in a non-deterministic world one would need stochastic calculus, which would give rise to second-order terms. In both (1.7) and its special case (2.3) these second-order terms are missing.
14. Multiply both sides of the first order condition $\partial u_k(.)/\partial c_{kj} = \lambda_k(t)\, p_{kj}(t)$ by $c_{kj}(t)$ and sum across the goods j:

    $$\sum_j \partial u_k(.)/\partial c_{kj}\, c_{kj}(t) = \lambda_k(t) \sum_j p_{kj}(t)\, c_{kj}(t)$$

    The factor $\sum_j p_{kj}(t)\, c_{kj}(t)$ on the right hand side equals total consumption spending, $C_k(t)$; and, because $u_k(.)$ is linear homogenous, the left hand side equals $u_k(t)$ or $v_k(t)$. Thus, $\lambda_k(t) = v_k(t)/C_k(t) \equiv \Pi_k(t)^{-1}$.
15. The exchange rate equation in Stulz (1987) also contains interest rate terms because in his model money is an argument of the utility function.
16. Balassa and Samuelson argue that the relative prices in (2.10) are determined by relative production costs, and hence, by relative productivities in the sectors producing traded and non-traded goods. Non-tradables are associated with services and tradables with industrial goods; and in a more developed economy the weight for services is larger than it is in a less developed economy. Deviations from absolute PPP are then explained as follows. The productivity of labor in the industrial sector relative to that in the service sector is higher the more developed the economy. Thus, if country 2 is the more developed country, then in country 2 the relative price of non-tradables versus tradables is higher than in country 1. If there are equal weights across countries, this produces a real exchange rate in excess of unity; and this conclusion holds a fortiori if the weight for services is higher in the richer country. Similarly, it is argued that, over time, relative prices of services tend to rise everywhere; however, as the weight for services is higher in the richer country, this effect leads to an appreciation of the real value of the currency of the more developed country, country 2.
17. In practice, tests of PPP have often relied on standard regression analysis of first-differenced data. The implications of our general model for such tests of PPP are discussed in Sercu et al. (1995), except for obvious generalizations associated with differences in risk aversions, time-varying parameters, and deviations between average and marginal inflation rates.
18. Other economies had severe exchange controls for a substantial part of the sample period (France, Italy, Spain, Scandinavia, all NICs and LDCs), suffered from missing data (Belgium). It is true that the UK, and especially also Japan, had exchange controls until the early 80s, but it was felt that these two major exchange rates could not be excluded from the tests. Dropping Japan from the tests does not affect the conclusions. We also used money supplies to proxy for aggregate consumption, which allows us to use monthly rather than quarterly observations. The drawbacks are that one then is assuming a cointegration relation between a country's spending and its money supply, which weakens the power of the test. The results with money are weaker than the ones reported here.
19. Note that, unlike in the Abuaf and Jorion (1990) tests, in Table 1 the hypothesis of a unit root cannot be rejected even for the real exchange rates. The likely cause of this difference is the loss of power from our use of a relatively short sample of quarterly data, rather than the long series of annual data used by Abuaf and Jorion but which then includes the time period where exchange rates were fixed rather than floating and there were restrictions on capital flows.
20. This holds whether we use data per country pair or all data at once, and, in the latter case, also whether we impose the restrictions one by one or all at a time
21. Relative to Nessén (1994), our cointegration analysis uses more recent results on critical values when testing for the number of cointegration relationships, and includes a

correction for small samples.

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